

Title: Kangaroo Conundrum: A Study Of A Quadratic Function.

Brief Overview:

This unit uses game theory to explore a quadratic function. Students will play a game to collect data and use this data to find a pattern for game outcomes. They will create numeric and graphic models to describe the pattern they discover while playing the game.

This unit proceeds with the use of scatter plots, linear and quadratic regressions using the graphing calculator. To find the quadratic function, the matrix package on the graphing calculator can be used to solve a system of equations. At the teacher's discretion, the quadratic function can be explored graphically using analytical geometry techniques. Using this model, students will be able to predict future outcomes of this game, explore the impact of modifications of this game, and extend these modeling techniques to other games and possible real-world situations.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

Number and Operations

The students will be able to understand the relationship between pairs of numbers, ways of representing numbers and patterns in numbers.

Algebra

Students will be able to represent and analyze situations using algebraic symbols. They will be able to use mathematical models to represent a physical situation and analyze changes in the situation.

Geometry

Students will be able to specify locations and describe special relationships using coordinate geometry.

Data Analysis and Probability

The students will understand scatter plots and use them to display data. They will determine regression equations and discuss the appropriateness of these equations as they relate to their data.

- **Process Standards**

- Problem Solving, Reasoning and Proof, Communication, Connections and Representation**

- Each process standard is integrated in this unit. By working in partners to find a model for the game, students will be required to exercise and improve their skills in doing mathematics, thinking about mathematics, understanding mathematics and communicating mathematics.

Links to Maryland High School Mathematics Core Learning Units:

Functions and Algebra

- **1.1.1**

- The student will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

- **1.1.2**

- The student will represent patterns and functional relationships in a table, as a graph, and/or by mathematical expression.

- **1.2.4**

- The student will describe how the graphical model of a non-linear function represents a given problem and will estimate the solutions.

- **1.2.5**

- The student will apply formulas and use matrices to solve real-world problems.

Geometry, Measurement, and Reasoning

- **2.1.3**

- The student will use transformations to move figures, create diagrams, and demonstrate geometric properties.

- **2.1.4**

The student will validate properties of geometric figures using appropriate tools and technology.

Data Analysis and Probability

- **3.2.1**

The student will make informed decisions and predictions based upon the results of simulations and data from research.

- **3.2.2**

The student will make predictions by finding and using a line of best-fit, and by using a given curve of best-fit.

Grade/Level:

Grades 9-12 /Algebra I & II and Analytical Geometry.

Duration/Length:

Depending on the extent to which the graphing calculator is applied to the unit, or whether the analytical geometry concepts are included, the unit should take between two and three classroom periods (90 minutes each) and nightly homework. Extra time may be necessary to instruct some students in the use of the graphing calculator.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Using basic knowledge of linear and quadratic equations.
- Inputting data into the calculator and creating scatter plots.
- Performing regressions and analyzing their appropriateness
- Using basic knowledge of matrix operations on the calculator.
- Using basic knowledge of the construction and terminology associated with parabolas will be necessary for the analytical geometry extension of this unit.

Student Outcomes:

Students will:

- Collect data to make a scatter plot.
- Determine the most appropriate regression equation, using a graphing calculator, to fit the data.
- Use the regression equations to make predications.
- Identify the characteristics of a parabola.

- Compare changes in the derived standard form of the equation with the modifications in the graph.
- Explore how changing the game rules would affect the model.

Materials/Resources/Printed Materials:

- Student work sheets
- Graphing calculators
- Choose one type of game piece: Chips/M & M's/Hershey kisses

Development/Procedures:

Day 1:

The teacher will introduce the lesson by demonstrating one or two rounds of the game on the overhead or have students serve as game pieces to demonstrate the game. The students will play the game completing rounds three and four while filling out their data chart. The students will look for a pattern to complete the data chart. Using the graphing calculator students will construct the scatter plot and check for Linear Regression. Students will comment on the inappropriateness of the linear model.

Day 2:

Students will refine their mathematical model by comparing the linear model with a quadratic model. The quadratic function will be determined by using matrices (Algebra II) and/or quadratic regression on the graphing calculator. Students will make predictions using the quadratic function. Students will evaluate the graph in terms of minimum or maximum, line of symmetry, x -intercepts and y -intercept. Discuss where this data analysis strategy could be applied to real-world situations.

Day 3:

Algebra I • Assessment

Algebra II • Students will put the quadratic equation into standard form, by completing the squares. Students will determine the focus, directrix, and length of the latus rectum for the parabola. Students will perform translations and transformations of the parabola.

Day 4:

Algebra II • Assessment.

Assessment:

This unit will include an assessment consisting of selected responses, student-produced responses, and brief constructed responses. The tasks will assess how well students understand how to evaluate data and find an appropriate mathematical model to make predictions. The assessment will demonstrate the students' ability to associate the concepts learned with quadratic functions and their transformations.

Assessment tools:

Graphing calculator

Administering the Assessment:

Time required: 45 minutes

Extension/Follow Up:

Students will be asked to modify the game rules and see the effect on the quadratic equation. Students can perform data analysis and find regression pairs for the game “Tower of Hanoi”. The game is explained with solutions on the following web sites:

<http://www.math.toronto.edu/mathnet/games/towermath.html>

<http://mathforum.org/dr.math/faq/faq.tower.hanoi.html>

Authors:

Thomas C Banford
Hickory High School
Chesapeake, Virginia 23322

Amina Mathias
Perryville High School
Cecil County, Maryland 21903

Carol A Oehlbeck
Greece Arcadia High School
Monroe County, New York 14612

Lynnette E Roller
Dulaney High School
Baltimore County, Maryland 21093

The Kangaroo Conundrum Day 1

Directions: You are going to play a game with a partner; the game requires one person to move the game pieces while the other keeps a record of the minimum number of moves required to complete the game. The object of the game is to move different colored game pieces placed at opposite ends of a strip of squares, to the respective opposing side. The allowed moves include slides and hops. The game pieces (kangaroos) starting on the left can only move right and vice versa. Kangaroos can jump other kangaroos of the opposing color. Any number of the opposing color kangaroos, which sit in adjacent squares to each other, may be jumped in a single move.

Determine the minimum number of moves needed for one, two, three, and four kangaroos of each color.



Number of moves _____



Number of moves _____

Use the strip provided to play the game two more times, first with three kangaroos of each color, then four of each color and record the minimum number of moves required for each.

Fill in the chart below using the pattern you found in the first four games.

Number of Kangaroos on one side	1	2	3	4	5	6	7	8	9	10	...	16
Minimum # of moves											...	

Now enter this data into the lists of the graphing calculator and prepare a scatter plot.

Does the scatter plot show a pattern? _____

If yes, what type of function do you think would best fit the data? Explain why you chose this type of function.

Write an equation to give the minimum number of moves required based on the number of kangaroos of one color. (Hint: You may use the statistics package on the graphing calculator.)

How did you arrive at this equation?

Enter the equation into the graphing calculator and graph your best-fit equation with your scatter plot. Now compare the table of values in the calculator with the data which you collected. Do you have a good model? Identify any problems with your model.

Homework: The rules of the game have been changed!!! Kangaroos can now only jump the opposing color ***one*** at a time (no jumps of 2 or 3 opponents). Kangaroos are now allowed to move ***either forward or backward***.

Directions: Play the game with up to and including four kangaroos on each side and record the results below.

Number of Kangaroos on one side	1	2	3	4	5	6	7
Minimum # of moves							

Is the model for this new game still a quadratic equation? Justify your answer.

Teacher's Guide

Day 1

Introduction: This unit asks the student to find a mathematical model (equation) that allows them to predict outcomes for a game. The prediction model is quadratic. Students should consider the simplest model (a line) and then reject this model since it will not accurately predict game outcomes in favor of a quadratic model.

Included in this guide is the strip worksheet needed to play the game. The teacher will have to decide what type of playing pieces they wish to use. Life Savers and Hershey Kisses work well. Teacher may model the game on the overhead or with students in the front of the class.

Solutions:

Number of moves for one kangaroo on a side – 3

Number of moves for two kangaroos on a side – 8

Number of Kangaroos on one side	1	2	3	4	5	6	7	8	9	10	...	16
Minimum # of moves	3	8	15	24	35	48	63	80	99	120	...	288

Now enter this data into the lists of the graphing calculator and prepare a scatter plot.

Does the scatter plot show a pattern? *Yes- odd numbers starting with 5 are added.*

If yes, what type of function do you think would best fit the data? Explain why you chose this type of function.

Students can come up with various equations. Explain why a linear equation is not the best choice because it does not accurately predict game outcomes. A quadratic function works the best.

Write an equation to give the minimum number of moves required based on the number of kangaroos of one color. (Hint: You may use the statistics package on the graphing calculator.)

$$y = n^2 + 2n$$

How did you arrive at this equation?

You can use the statistics package on a calculator, entering the values in two lists and then asking the calculator to calculate regression curves. You can also translate the basic equation: $y = n^2$ into the equation $y = (n+1)^2 - 1$ from the scatter plot.

Enter the equation into the graphing calculator and graph your best-fit equation with your scatter plot. Now compare the table of values in the calculator with the data which you collected. Do you have a good model? Identify any problems with your model.

If the student gave the correct quadratic model, they should be able to predict outcomes accurately. If they have a different model, they should answer that the model is not good because the model does not accurately predict game outcomes.

Homework: The rules of the game have been changed!!! Kangaroos can now only jump the opposing color ***one*** at a time (no jumps of 2 or 3 opponents). Kangaroos are now allowed to move ***either forward or backward***.

Directions: Play the game with up to and including four kangaroos on each side and record the results below.

Number of Kangaroos on one side	1	2	3	4	5	6	7
Minimum # of moves	3	8	19	36	59	88	123

Students can look at common differences or put the data into the statistics editor of a graphing calculator. The model for the new game is basically quadratic but the value for two kangaroos seems to be problematic.

Materials/Resources:
Game strip for Day1

Kangaroo Conundrum

Day 2

Find the appropriate model

Background: Recall that we previously collected data from our game and prepared a scatter plot. We further determined that a linear equation was not the best model for the data. What other types of functions could we consider for this model?

Complete this table to determine if the differences are constant.

A linear function is indicated by a constant first difference. (This means that the difference between any two consecutive rows will be the same). A linear function is a first degree equation. What type of function do you think is indicated by a constant second difference?

Number of kangaroos on one side	Minimum number of moves	1 st difference	2 nd difference
1	3	-	-
2	8	5	-
3	15	7	2
4	24		
5			
6			
7			
8			
9			
10			

One way to determine the equation for this model is to use matrices on the graphing calculator.

Consider the general form of a quadratic equation.

$$ax^2 + bx + c = 0$$

Matrices can be used to solve systems of equations.

Using the data from our table, write three equations with three variables.

$$\text{For } x=1, a + b + c = 3$$

$$\text{For } x=2, 4a + 2b + c = 8$$

$$\text{For } x=3, 9a + 3b + c = 15$$

Use the coefficients of the variables a , b , c from the three equations to create the matrix:

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}$$

Use the first three values from the data table to form a second matrix.

$$\begin{bmatrix} 3 \\ 8 \\ 15 \end{bmatrix}$$

Enter these matrices into the matrix package of the graphing calculator.

In order to solve three equations in three variables, the inverse of the 1st matrix is multiplied by the 2nd matrix to determine the coefficients of the quadratic equation.

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 3 \\ 8 \\ 15 \end{bmatrix}$$

What is the resulting matrix?

Since these values are the coefficients of the terms of the quadratic equation in general form, write the resulting quadratic equation.

In order to check to see if this quadratic equation matches the model, play the game using four kangaroos on one side, and see if the minimum number of moves agrees with the minimum number of moves from the data table, which is 24.

This quadratic equation can be used to find the minimum number of moves for 50 kangaroos on one side.

What is the minimum number of moves for 50 kangaroos on one side?

The homework assignment generated new data. Use differences columns to determine the degree of the equation. If it is quadratic, use the homework data and the matrix method to write a quadratic equation for this new model.

Show your work for the differences, matrices and the equation used to find the coefficients, as well as the resulting quadratic equation.

Find the minimum number of moves for 50 kangaroos.

How does the result compare with the result for the previous model?

Teacher's Guide Day 2

The students should realize that the model does not fit a linear equation. They may suggest that the model could be quadratic or exponential. The use of difference columns is one method that can be used to determine if the model is quadratic. The lesson will begin by completing the data table. The purpose is to find where, if at all, the differences become constant. Notice that the 2nd column shows a common difference of 2. This shows that the equation is quadratic.

Number of kangaroos on one side	Minimum number of moves	1 st difference	2 nd difference
1	3	-	-
2	8	5	-
3	15	7	2
4	24	9	2
5	35	11	2
6	48	13	2
7	63	15	2
8	80	17	2
9	99	19	2
10	120	21	2

In response to the question: What type of function do you think is indicated by a constant second difference? The answer is *quadratic*.

The resulting matrix is $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$. The resulting equation is $y = x^2 + 2x$.

The minimum number of moves for 50 kangaroos is 2,600.

Extension/Follow Up:

Students will be asked to modify the game rules and see the effect on the quadratic equation. Students can perform data analysis and find regression pairs for the game "Tower of Hanoi". The game is explained with solutions on the following web sites:

<http://www.math.toronto.edu/mathnet/games/towermath.html>

<http://mathforum.org/dr.math/faq/faq.tower.hanoi.html>

Kangaroo Conundrum (Day 3)

Evaluating our Quadratic Equation

The quadratic equation which we derived as the model for our data was $y = x^2 + 2x$. This is the general form of the quadratic equation. We can also put this equation into standard form for a parabola. This equation can then be graphed, on the graphing calculator, in either general or standard form. We can use the completing the square method to put this equation in standard form for a parabola. In standard form this equation would be written as

_____.

With the equation in standard form, specify the following information:

- a. direction of opening _____
- b. coordinates of the vertex _____
- c. equation of the axis of symmetry _____
- d. roots of the quadratic equation _____
- e. coordinates of the focus _____
- f. equation of the directrix _____
- g. length of the latus rectum _____

Now rewrite the standard form equation in such a fashion that the vertex would be moved two units up and two units to the right on the coordinate plane and the direction of opening would be changed to down.

Kangaroo Conundrum (Day 3)
Homework

This data was collected with the modified rules to the game. Find a model for this set of data by using a quadratic regression and specify the resulting quadratic equation.

Number of kangaroos of one color	1	2	3	4
Minimum number of moves required	3	8	19	36

The equation is:

Now graph both the original quadratic equation model and the quadratic equation model for the game with the revised rules. Then describe the relationship between the two graphs. Determine the points of intersection and relate these coordinates back to the game. Speculate on why the graphs are related in this manner.

Teachers' Guide

Kangaroo Conundrum (Day 3)

Introduction: Day three involves applying analytical geometry concepts to the quadratic model derived from the game data. The students will be asked to change the quadratic equation from general form to the standard form for a parabola by the completing-the-square method. With the equation in standard form, the students will be tasked with determining the characteristics of the resulting parabola. The students will further be required to graph the equation on a graphing calculator. Lastly, the students will be asked to move the vertex of the parabola three units up and three units to the right on the coordinate plane and to change the direction of opening from up to down. The students will then rewrite the equation in standard form to accomplish this translation/transformation.

Algebra I students may complete part of this activity by graphing the equation in general form on a graphing calculator. They can then analyze the graph of the parabola to determine the vertex, axis of symmetry, and direction of opening.

Procedures and solutions (Algebra I and Algebra II):

The students should change the quadratic equation derived from the game data into a standard form equation for a parabola. The equation in standard form would be written as follows: $y = (x + 1)^2 - 1$. The students should graph the equation in both general and standard form on the graphing calculator.

The vertex can be identified from either the graph or from the standard form equation as $(-1, -1)$. The parabola can be seen to open upward. The equation of the axis of symmetry is the x -coordinate of the vertex and is $x = -1$. The equation of the axis of symmetry can also be found from the formula $x = -b/2a$. Students should be encouraged to verify the coordinates of the vertex by looking in the table of values in the calculator. The students should clearly see the symmetry of the parabola in the table of values. The students should also determine the roots of the equation from the x -intercepts of the graph as 0 and -2. These roots may also be found by factoring or by the quadratic formula. Students should be encouraged to discuss the significance of these values to the game and should identify the restriction on the domain in the game.

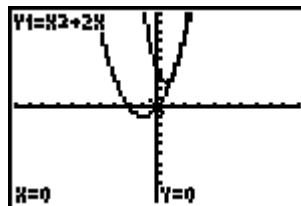
Procedures and solutions (Algebra II):

The students should be aware that only the value of a , the coefficient of the quadratic term, and the coordinates of the vertex are required to write the standard form equation of a parabola. Likewise, the value of a , is required to determine the coordinates of the focus and the equation of the directrix. The distance from the vertex to the focus along the axis of symmetry and the distance from the vertex to the directrix along the axis of symmetry are given by the ratio 1 divided by $4a$. This distance for the given parabola is $\frac{1}{4}$. Thus, the coordinates of the focus are $(-1, -1.75)$ since the focus lies inside the parabola and above the vertex. The equation of the directrix then is $y = -1.25$ as it is a horizontal line outside the parabola which intersects the axis of symmetry one-fourth of a unit below the vertex. Students should be reminded that the focus and directrix are used to construct the parabola. Lastly, the length of the latus rectum should be determined as the absolute value of the ratio of $1/a$. In the given parabola, this length is 1 and would reflect the whether the parabola opens quickly or slowly.

In conclusion, the students are asked to rewrite the standard form equation of the parabola to accomplish a translation and transformation of the graph. The revised equation is $y = -(x-2)^2 + 2$. The students should graph each equation to verify the required translation and transformation.

Homework solutions (Algebra II):

The quadratic equation for the game with modified rules turns out to be $y = 3x^2 - 4x + 4$. This equation was determined by a quadratic regression of the homework data. The points of intersection of the graphs of the two modeling equations are $(1, 3)$ and $(2, 8)$ which were the identical results for the first two rounds of both games. The modification of the rules caused the results to diverge commencing with the third round. Students will have varying ideas as to why the graphs have the resulting relationship between them, and there will be no **right** answer.



Assessment

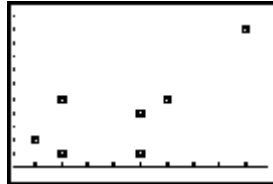
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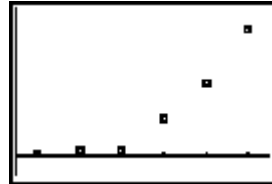
Selected Response

1. Which of the following is a scatter plot of a linear function?

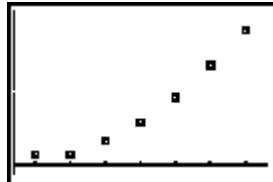
A.



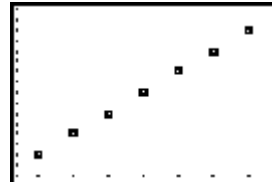
B.



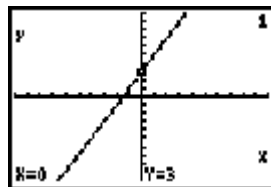
C.



D.



2. Which of the following equations match this graph?



- A. $y = -2x + 3$
B. $y = -2x^2 + 3$
C. $y = 2x + 3$
D. $y = 2x^2 - 3$
3. Which of the following equations represents a quadratic equation?
A. $y = 2x^3$
B. $y = 2x^2$
C. $y = 2x^4$
D. $y = 2x$
4. What is the next number in this pattern? 1, 7, 13, 19, 25, ...
A. 31
B. 32
C. 33
D. 34

5. Which equation gives the next number in the pattern?

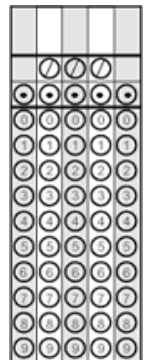
x	y
1	3
2	9
3	19
4	33
5	51
6	73
7	99

- A. $y = x^2 + 1$
- B. $y = 2x^2 + 1$
- C. $y = x^2 + 2$
- D. $y = 2x^2 + 2$

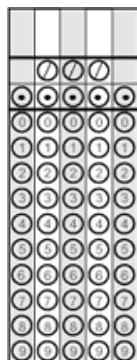
Student Produced response

6. In a game called Splat, a good student can win a game with the given number of playing pieces in the number of moves displayed in the chart below. What is the required number of moves the student must make in order to win at Splat if they play with 10 pieces?

Number of playing pieces	# of moves required to win
1	4
2	7
3	12
4	19
5	28



7. Find the coefficient of the x term in the quadratic equation which passes through (1, 5), (3, 9) and (2, 8). (*Algebra II students or above*)

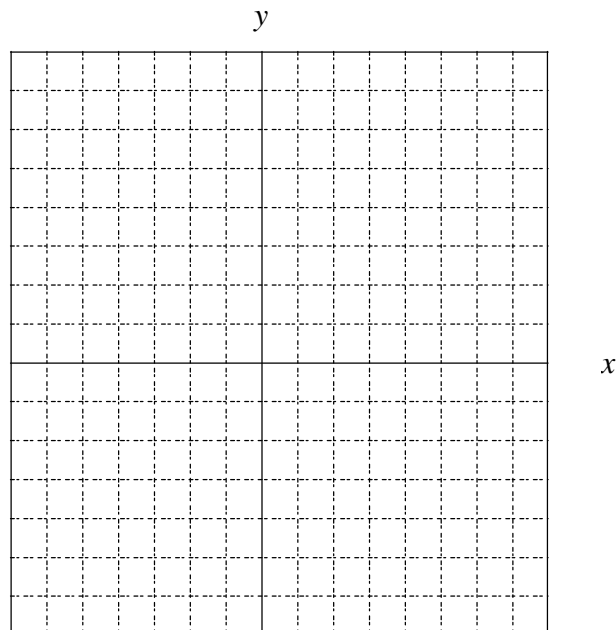


8. Find the x -coordinate of the vertex for the quadratic $y = x^2 - 5x + 9$.

	7	7	7	
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

Brief Constructed Response:

9. A group of students gives the answer to the Kangaroo Conundrum as $y = x^2 - 4x + 4$, where y is the number of moves required to win and x is the number of game pieces on one side of the board. Explain using words and a graph, why this group must be wrong.



10. A ball thrown by an outfielder travels in an arc modeled by the equation $y = -0.002x^2 + 0.4x + 6$, where distances are measured in feet. (*Analytical Geometry*)
- a. Find the axis of symmetry.

- b. Find the coordinates of the vertex.

- c. Does the graph have a *maximum* or a *minimum*? Explain your response using words, symbols or both.

11. a. Put this equation $-24 = 2x - x^2$ in standard form by completing the square.

- b. Find the coordinates of the focus.

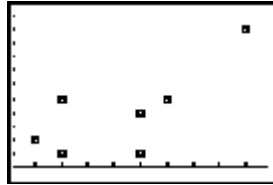
- c. Write the equation of the directrix.

Assessment-Answer key

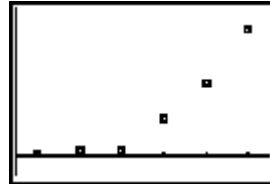
Selected Response

1. Which of the following is a scatter plot of a linear function? **Answer: D**

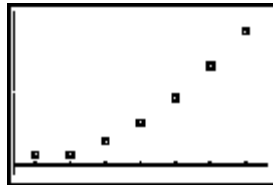
A.



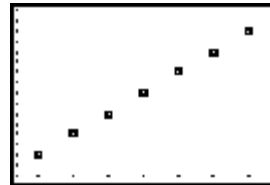
B.



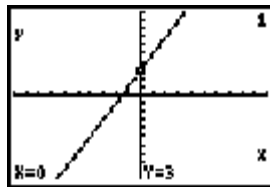
C.



D.



2. Which of the following equations match this graph? **Answer: C**



A. $y = -2x + 3$

B. $y = -2x^2 + 3$

C. $y = 2x + 3$

D. $y = 2x^2 - 3$

3. Which of the following equations represents a quadratic equation? **Answer: B**

A. $y = 2x^3$

B. $y = 2x^2$

C. $y = 2x^4$

D. $y = 2x$

4. What is the next number in this pattern? 1, 7, 13, 19, 25, ... **Answer: A**

A. 31

B. 32

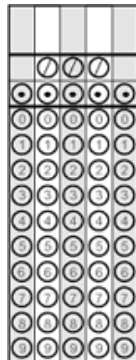
C. 33

D. 34

5. Which equation gives the next number in the pattern? **Answer: B**

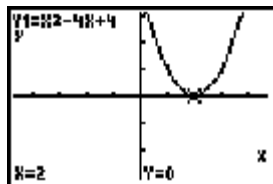
x	y	
1	3	
2	9	
3	19	
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8. Find the x -coordinate of the vertex for the quadratic $y = x^2 - 5x + 9$. **Answer: 2.5**



Brief Constructed Response:

9. Suppose another group of students gives the answer to the Kangaroo Conundrum as $y = x^2 - 4x + 4$, where y is the number of moves required to win and x is the number of game pieces on one side of the board. Explain using words and a graph, why this group must be wrong.



This graph shows that for 2 game pieces, the number of moves required to win is zero. (Vertex is 2, 0) This is not possible.

Grading rubric:

3 points- The graph is correct and the written answer explains the solution referring to the graph.

2 points – the graph is incorrect but the written solution is basically correct.

1 point – The graph is correct but the written solution is incorrect.

10. A ball thrown by an outfielder travels in an arc modeled by the equation $y = -0.002x^2 + 0.4x + 6$, where distances are measured in feet. (*Analytical Geometry*)

a. Find the axis of symmetry.

$$a = -0.002 \quad b = 0.4 \quad x = -b/2a$$

Substitute the values and the solution will be $x=100$.

b. Find the coordinates of the vertex.

Take the value $x=100$ and substitute it into the given equation $y = -0.002x^2 + 0.4x + 6$, which will give $y = -0.002(100)^2 + 0.4(100) + 6 = 26$. Therefore, the coordinates of the vertex is $(100, 26)$.

c. Does the graph have a *maximum* or a *minimum*? Explain using words, symbols or both.
The graph has a minimum because the coefficient of the x^2 term is negative, which is -0.002 .

11. a. Put this equation $-24 = 2x - x^2$ in standard form by completing the square.

Step-wise solution: $-x^2 + 2x = -24$

Multiply through with negative 1 $x^2 - 2x = +24$

Take $\frac{1}{2}$ the co-efficient of the linear term, square it and add it to both sides

$$x^2 - 2x + 1 = 25$$

Now factor the left side. Since it is a perfect square, it will give us

$$(x - 1)^2 = 25 \quad y = (x - 1)^2 - 25 \quad \text{This is standard form)}$$

Take the square root of both sides, we get

$$x - 1 = \pm 5$$

Solving the equations,

$$x - 1 = 5 \text{ and } x - 1 = -5$$

The solutions are,

$$x = 6 \text{ and } x = -4$$

b. Find the coordinates of the focus.

From standard form the vertex can be found as $(1, -25)$

Use the formula: $1/4a$ is the distance of the vertex to the focus along the axis of symmetry.

With $a = 1$ the distance is $\frac{1}{4}$, the x -coordinate does not change since we are moving along the axis of symmetry, so the coordinates the focus are $(1, -24.75)$, since the focus lies $\frac{1}{4}$ of a unit above the vertex.

c. Write the equation of the directrix.

With the information in part b the equation of the directrix is determined by moving $\frac{1}{4}$ of a unit below the vertex. Therefore the equation of the directrix is $y = -25.25$